Name: Netty the Incredible

Instructor: ______________________

Math 10170, Exam I,
March 4, 2015

- The Honor Code is in effect for this examination. All work is to be your own.
- You may use your Calculator.
- The exam lasts for 50 minutes.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 10 pages of the test.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	(a)	(b)	(c)	(d)	(e)
2.	(a)	(b)	(c)	(d)	(e)
3.	(a)	(b)	(c)	(d)	(e)
4.	(a)	(b)	(c)	(d)	(e)
5.	(a)	(b)	(c)	(d)	(e)
6.	(a)	(b)	(c)	(d)	(e)
7.	(a)	(b)	(c)	(d)	(e)

Please do NOT write in this box.

Multiple Choice	______
8.	______
9.	______
10.	______
11.	______
12.	______
Total	______

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Multiple Choice

1.(6 pts.) Ten Judges have ranked the performance of 4 gymnasts, Mary, Jane, Katy and Ann

# of Voters

	3	**3**	**4**	Ave.
Mary	1	2	4	$(3+6+16)/10 = \frac{25}{10} = 2.5$
Jane	2	3	1	$(6+9+4)/10 = 19/10 = 1.9$
Katy	3	4	3	$(9+12+12)/10 = 33/10 = 3.3$
Ann	4	1	2	$(12+3+8)/10 = 23/10 = 2.3$

The winner using the Borda Method is:

(a) Ann

(b) Mary

(c) Katy

(d) Jane

(e) Tie for first place between Mary and Ann

2.(6 pts.) Consider the following matrices:

$$A = \begin{pmatrix} 2 & 0 & 1 \\ 1 & 2 & -1 \end{pmatrix} \quad B = \begin{pmatrix} 0 & 2 & 1 \\ 4 & 1 & 0 \end{pmatrix} \quad C = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}.$$

Which of the following matrices is equal to $(A - B)C$?

(a) $\begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$. (b) $\begin{pmatrix} -2 \\ -2 \end{pmatrix}$. (c) $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$.

(d) $\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$. (e) $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$.

$$\underbrace{\begin{pmatrix} 2 & -2 & 0 \\ -3 & 1 & -1 \end{pmatrix}}_{(A-B)} \underbrace{\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}}_{C} = \begin{pmatrix} 2\cdot 1 + (-2)\cdot(2) + (0)(1) \\ (-3)(1) + (1)(2) + (-1)(1) \end{pmatrix} = \begin{pmatrix} -2 \\ -2 \end{pmatrix}$$

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3.(6 pts.) At the Middle Earth games there are 9 events and 4 participating teams, The Hobbits, The Dwarves, The Elves and the Giants.

Number of Events = 9.

	1	**2**	**2**	**1**	**2**	**1**
Dwarves	1	4	2	3	2	4
Hobbits	4	1	1	4	4	1
Elves	2	3	3	1	1	3
Giants	3	2	4	2	3	2

Each year "The Grand Prize" is awarded to the team with the best overall performance. If a Condorcet winner exists, the Grand Prize is awarded to that team, otherwise a Condorcet completion process is used to decide the winner. Which of the following is true?

H v D → H

5 v 4

(a) The Hobbits are the Condorcet winner

(b) The Giants are the Condorcet winner

H v E → H

5 v 4

(c) The Dwarves are the Condorcet winner

(d) The Elves are the Condorcet winner

H v G → H

5 v 4

(e) There is no Condorcet winner

4.(6 pts.) How many games must be played in a round robin tournament with 10 teams, where each team plays every other team exactly once.

(a) 20 (b) 55 (c) 45 (d) 50 (e) 10

$$\frac{n(n-1)}{2} = \frac{10(9)}{2} = 45$$

$n = 10$

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5.(6 pts.)The following statistics on passing for quarterback Peyton Manning show his completion record for the 2014 season. (CMP = completed, INC = Incomplete)

	CMP	INC	Total
Home Games	184	78	262
Away Games	211	124	335
TOTAL	395		

$P(C|H) = \frac{\# C \cap H}{\# H} = \frac{184}{262}.$

If we choose a pass at random from the records, let H be the event that it was in a home game and let C be the event that it was complete, Which of the following statements are true?

(a) $P(C|H) = \frac{345}{397}$ (b) $P(H|C) = \frac{184}{597}$ (c) $P(H|C) = \frac{262}{395}$

(d) $P(C|H) = \frac{184}{262}$ (e) $P(C|H) = \frac{262}{395}$

$P(H|C) = \frac{\# C \cap H}{\# C} = \frac{184}{395}.$

6.(6 pts.) An experiment consists of flipping a coin until a tail appears. As soon as a tail appears, the experimenter stops and the experimenter records the sequence of heads and tails. What is the sample space for this experiment?

STOP AS SOON AS A TAIL APPEARS.

(a) $\{T, HT, HHT, HHHT, HHHHT, \ldots\}$

(b) $\{H, T\}$

(c) $\{H, T, H, T, H, T\}$

(d) $\{H, TH, THHH, THHHH, \ldots\}$

(e) $\{HHH, HHT, THH, TTT, THT, HTH, HTT, TTH\}$

(The Trial ALWAYS ends WITH THE Appearance of THE FIRST TAIL. ALTHOUGH UNLIKELY THE Coin could be Flipped A MILLION times OR MORE before you see the 1st Tail. The S. S. is INFINITE)

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l_i = # losses team i

W_i = # wins Team i

7.(6 pts.) The following table shows the results of a round robin in progress (up to Feb. 25 2015 the Six Nations Championship in Rugby).

$b_i = 1 + \frac{W_i - l_i}{2}$		Ireland	England	Wales	Scotland	France	Italy	W_i	l_i
2	Ireland					18-11	26-3	2	0
2	England			21-16			47-17	2	0
1	Wales		16-21		26-23			1	1
0	Scotland			23-26		8-15		0	2
1	France	11-18			15-8			1	1
0	Italy	3-26	17-47					0	2

$$b_i = 1 + \frac{W_i - l_i}{2}$$

Which of the following matrix equations must be solved in order to find the Colley Ratings (keeping the same ordering of the teams as above)?

2 + # games played

− # Times Team 1 played Team 2

$$\begin{bmatrix} 2+g_1 & -n_{12} & & \\ -n_{12} & 2+g_2 & & \\ & & \ddots & \\ & \cdots & & 2+g_6 \end{bmatrix} \begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ r_6 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \\ b_6 \end{bmatrix}$$

Colley Matrix

(a)
$$\begin{pmatrix} 4 & 0 & 0 & 0 & -1 & -1 \\ 0 & 4 & -1 & 0 & 0 & -1 \\ 0 & -1 & 4 & -1 & 0 & 0 \\ 0 & 0 & -1 & 4 & -1 & 0 \\ -1 & 0 & 0 & -1 & 4 & 0 \\ -1 & -1 & 0 & 0 & 0 & 4 \end{pmatrix} \begin{pmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ r_6 \end{pmatrix} = \begin{pmatrix} 30 \\ 35 \\ -2 \\ -10 \\ 0 \\ -53 \end{pmatrix}$$

(b)
$$\begin{pmatrix} 2 & 0 & 0 & 0 & -1 & -1 \\ 0 & 2 & -1 & 0 & 0 & -1 \\ 0 & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & -1 & 2 & -1 & 0 \\ -1 & 0 & 0 & -1 & 2 & 0 \\ -1 & -1 & 0 & 0 & 0 & 2 \end{pmatrix} \begin{pmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ r_6 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

(c)
$$\begin{pmatrix} 4 & 0 & 0 & 0 & -1 & -1 \\ 0 & 4 & -1 & 0 & 0 & -1 \\ 0 & -1 & 4 & -1 & 0 & 0 \\ 0 & 0 & -1 & 4 & -1 & 0 \\ -1 & 0 & 0 & -1 & 4 & 0 \\ -1 & -1 & 0 & 0 & 0 & 4 \end{pmatrix} \begin{pmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ r_6 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

(d)
$$\begin{pmatrix} 2 & 0 & 0 & 0 & -1 & -1 \\ 0 & 2 & -1 & 0 & 0 & -1 \\ 0 & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & -1 & 2 & -1 & 0 \\ -1 & 0 & 0 & -1 & 2 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ r_6 \end{pmatrix} = \begin{pmatrix} 30 \\ 35 \\ -2 \\ -10 \\ 0 \\ 0 \end{pmatrix}$$

(e) None of the above

Each team has played A TOTAL of 2 games

⇒ Diag entries are all 2+2 = 4.

So ans is (a) or (c).

Ireland has played Fr. & It

Team 1 " " T5 & T6

So The 1st row looks like

[4 0 0 0 −1 −1]

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Partial Credit

You must show your work on the partial credit problems to receive credit!

29W
21L.

8.(10 pts.) The following is a sequence of consecutive wins and losses for 50 games for the Milwaukee Brewers in the 2014 season:

W L L W W W W W W W W W L L W L W W W W L W W W L W W L L W L L
W L L L W W W L W W L L L L W L W L

The team won roughly 50% of their games in 2014.

If we choose a game at random from the 50 above,
let W we the event that the game chosen is a win,
let WP be the event that the game prior to the one chosen was a win,

(a) How many of the above outcomes are in WP?
(it might help to circle these outcomes in the data).

29 WP — $12\ L \cap WP$
$17\ W \cap WP$

(b) What is $P(W|WP)$?

$P(W|WP) = \frac{\# W \cap WP}{\# WP} = \frac{17}{29}$

(c) What is $P(W)$?

$P(W) = \frac{\# W}{\# \text{games}} = \frac{29}{50}.$

(d) Would you say that there is evidence of a hot hand effect in the data?

Is $P(W|WP) \underset{\text{significantly}}{>} P(W) \Rightarrow$ Is $\frac{17}{29}$ significantly larger than $\frac{29}{50}$

Is .586 sig. Larger than .58

Probably not

The evidence is not quite strong enough to suggest that a win is more likely if the previous game was won.

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9.(10 pts.)

$$\begin{aligned} 2x + y - z + 3w &= 8 \\ x - 10y + 4z + w &= 10 \\ 3x + y + z &= 5 \end{aligned}$$

(a) Write the above system of equations as a matrix equation $CX = D$.

$$\begin{bmatrix} 2 & 1 & -1 & 3 \\ 1 & -10 & 4 & 1 \\ 3 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 8 \\ 10 \\ 5 \end{bmatrix}$$

(b) Write the following system of equations as a matrix equation $AX = B$.

$$\begin{aligned} x + 2y &= 4 \\ 3x + 5y &= 3 \end{aligned}$$

$$\underbrace{\begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x \\ y \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} 4 \\ 3 \end{bmatrix}}_{B}.$$

(c) Solve the system in part (b) by finding the matrix A^{-1} and multiplying the equation by A^{-1}. (show your work for credit).

$$A^{-1} = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix} = \frac{1}{5-6}\begin{pmatrix} 5 & -2 \\ -3 & 1 \end{pmatrix}$$

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \qquad = -1\begin{pmatrix} 5 & -2 \\ -3 & 1 \end{pmatrix} = \begin{pmatrix} -5 & 2 \\ 3 & -1 \end{pmatrix}$$

$A^{-1} = \begin{pmatrix} -5 & 2 \\ 3 & -1 \end{pmatrix}$ Solution: $x =$ -14, $y =$ 9.

$$\begin{pmatrix} x \\ y \end{pmatrix} = A^{-1}A\begin{pmatrix} x \\ y \end{pmatrix} = A^{-1}\begin{bmatrix} 4 \\ 3 \end{bmatrix} = \begin{pmatrix} -5 & 2 \\ 3 & -1 \end{pmatrix}\begin{pmatrix} 4 \\ 3 \end{pmatrix}$$

$$= \begin{pmatrix} -20+6 \\ 12-3 \end{pmatrix} = \begin{pmatrix} -14 \\ 9 \end{pmatrix}$$

10

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10.(10 pts.) (a) What is the longest run of wins in the sequence of consecutive wins and losses for 50 games for the Milwaukee Brewers given below:

W L L W W W W W W W W W L L W L W W W W L W W W L W W L L W L L
W L L L W W W L W W L L L L W L W L

9.

(b) What is the expected length of the longest run of heads in a sequence of heads and tails generated by flipping a fair coin 50 times?

$\frac{\ln(25)}{\ln(2)}$ or $-\frac{\ln((0.5)k)}{\ln(.5)} = \frac{\ln(.25)}{\ln(2)} \cong 5.$

(c) Based on your results in parts (a) and (b), do you think it is likely that the probability of winning remained constant at 0.5 for the Milwaukee Brewers throughout this sequence of games?

No The longest run in the data is too long compared to what we would expect in a sequence generated randomly with a 50% chance of a win in every game.

Why is this a different result than that obtained in Q8? It isn't really a different answer! If we define "The hot hand" as the probability of a win increasing after one win, then Q8 shows there is no hot hand effect. However, if we define the "hot hand" as an increase in the probability of a win after 2 or more wins, we do see a hot hand effect.

We are testing for different things in Q8 and Q10

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11.(10 pts.) This problem appears as Problem 1 on the take home part of the exam. You may use this page for rough work.

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12.(18 pts.) This problem appears as Problem 2 on the take home part of the exam. You may use this page for rough work.